in our Curve, the greateſt Breadth is when the Point F *divides the Line* A B *in extream and mean Proportion: whereas in the* Foliate *it is when* A B *is triple in power to* B F. *And the greateſt* E F *or Ordinate in the* Foliate *is to that of our Curve nearly as* 3 *to* 4, *or exactly as* $\sqrt{\frac{2}{3}\sqrt{\frac{1}{3}}-\frac{1}{3}}$ *to* $\sqrt{5\sqrt{\frac{5}{4}}-5\frac{1}{2}}$.

But ſtill theſe Differences are not enough to make them two diſtinct Species, *they being both defined by a like Equation, if the* Aſymptote S G P *be taken for the Diameter. And they are both comprehended under the* fortieth Kind *of the* Curves *of the* third Order, *as they ſtand enumerated by Sir* Iſaac Newton, *in his incomparable Treatiſe on that Subject.*

IV. *An eaſy Mechanical Way to divide the Nautical Meridian Line in* Mercator's *Projection; with an Account of the Relation of the ſame Meridian Line to the* Curva Catenaria. *By* J. Perks, M. A.

THE moſt uſeful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd *Mercator*'s; tho' its true Nature and Conſtruction is ſaid to be firſt demonſtrated by our Countryman Mr. *Wright*, in his *Correction of the Errors in Navigation.* In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous,) but the Minutes and Degrees (or ſtrictly, the *Fluxions of the Meridian,*) at every ſeveral Latitude are proportional to their reſpective *Secants*. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the ſame Latitude is to *Radius*.

The

The Reason of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection, the Meridians being all parallel, a Degree of Longitude at (suppose) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but *half* as much, the Radius of the Parallel of 60 Deg. (that is its *Cosine*) being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to those of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Cosine*.

In *Fig.* 5. let the Radius CD represent half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant; then is CS equal to its *Cosine* : and $CS : CM :: CD (= CM) : CE$, that is, as Cosine : to Radius :: so is Radius : to Secant. The Cosines being then, in this Projection, suppos'd all equal to Radius, or (which comes to the same) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude, (by the precedent Analogy) is supposed equal to the Secant of Latitude; and consequently the Elements (Minutes, *&c.*) of the Meridian must be proportional to their respective Secants,

The Way Mr. *Wright* takes for making his Table of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and so proceeding to 89 Deg. Dr. *Wallis* (in *Phil. Transf.* No. 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$ &c. which gives the *Merid. part* required: How to find the same Mechanically by means of an easily-constructed Curve Line, is what I shall now shew.

1. Prepare a Rular *A B* (*Fig.* 2.) of a convenient Length, in which let *B o* be equal to the Radius of the intended Projection. To the Point *o* as a Center (on the narrower Edge of the Rular) fasten a little Plate-Wheel *w h* tight to the Rular, and of a Diameter a little more than the thickness of the Rular. Let *K R* (*Fig.* 3.) represent another long Rular, to which *A R* is a perpendicular Line. Place the Rular *A B* upon the Line *A R*, with the Center of the Wheel at *A*. Then with one Hand holding fast the Rular *K R*, with the other Hand slide the end *B* of the Rular *A B* by the Edge of *K R*; so will the little Wheel *w h* describe on the Paper a Curve Line *A C B*, to be continued as far as is convenient.

2. Having drawn the Curve *A C B*, draw a streight Line *K R* by the Edge of the Rular *K R*: which Line is the *Meridian* to be divided, and also an Asymptote to the Curve *A C B*.

3. In this Meridian, (accounting *R* to be the Point of its Intersection with the Equator,) the Point answering to any Degree of Latitude is thus found. In the perpendicular *A R*, make *R G* equal to the *Cosine* of Latitude (Radius being *A R*,) and from *G* draw *G C* parallel to *K R*, and intersecting the Curve in *C*. With Center *C* and Radius $CM = AR$, strike an Arc cutting the Meridian at *M*; so is *M* the Point desir'd.

4. In the Curve *A C*, let *c* be a Point infinitely near to *C*, and *c m*, ($= CM$,) a Tangent to the Curve at *c*, making the little Angle *M C m*, to which let the Angle *R A r* be equal: so is $Rr = Md$ (a Perpendicular from *M* to *c m*.) Draw *C D* equal and parallel to *A R*, intersecting *K R* in *S*. With Center *C* and Radius *C D* draw the Arc *D M*, and its Tangent *D E* and Secant *C E*.

5. Because of the like Triangles *C D E*, *M d m*; $CD : CE :: Md : Mm$, that is, as Radius to Secant of the Arc *D M*, (whose Cosine is $CS = GR$,) :: so is *M d*

 (=

(= *R r* a Degree or Particle of the Equator:) to *M m* the Fluxion or correſpondent Particle of the Meridian Line *R M*. Whence, and from what is premiſed concerning the Nature of this Nautical Projection, 'tis evident that *R M* is the *meridional Part* anſwering to the Latitude whoſe Coſine is *G R*. Or thus; With Center *R* and Radius *A R* deſcribe the Quadrant *A x α*, in which let the Arc *A x* be equal to the given Lat. From *x* draw *x C* parallel to *K R*, and interſecting the Curve in *C*, ſo is *C x* the Meridional Part deſir'd being equal to *R M*, as is eaſy to ſhew.

As to the other Properties of this Curve, tis evident, from its Conſtruction, that its *Tangent* (as *C M*) is a *Conſtant Line* every where equal to *A R*; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve *A C B* may, for diſtinction, be call'd the *Equitangential Curve*.

7. The Fluxion of the Area *A R M C* is the little Sector or Triangle *M C d*, which ſame is alſo the Fluxion of the Sector *C D M*: whence the Areas *A R M C*, *C D M* are equal, and the whole Area *A C B* &c, *K M R* being infinitely continued, is equal to the Quadrant *A R α*.

8. To find the Radius of Curvature of any Particle, as *C c*, from *C* draw an indefinite Line *C T* perpendicular to *C M*, (on the concave ſide of the Curve) and from *c* another Line perpendicular to *c m*, which Lines, (becauſe of the Inclination of *C M* to *c m*,) will ſomewhere meet as at *T*, making an Angle *C T c* = *M C m*. Theſe Angles being equal, their Radii are proportional to their Arcs: therefore, *M d* : *C c* :: *M C* : *C T*. But *C c* = *d m* (becauſe of *C M* = *c m*) ſo that *M d* : *d m* (:: *C D* : *D E*) :: *C M* : *C T*. But *C D* = *C M*, therefore *C T* = *D E* = Tangent of the Arc *D M*.

9.

9. So that ſuppoſing *ATt* a Curve Line in which are all the Centers of Curvature of the Particles of *ACB*, any point as *T* being found as before, the Length *AT* (by the nature of *Evolution of Curves*,) is every where equal to the *Tangent* of its correſpondent Circular Arc *DM*. The Point *T* is alſo found by making *MT* perpendicular to *RM*, and equal to the Secant *CE*: for ſo is the Angle $CMT = MCD$; and the Triangle *MCT* equal to the Triangle *CDE*.

10. Let *AHh* be an Equilater Hyperbola whoſe Semiaxis is *AR* and Center *R*. In the Meridian let *RP* be equal to the Tangent *DE*. Join *AP*, and draw $PH = AP$ and parallel to *AR*. Compleat the Parallelogram *HNRP*, ſo will the Point *H* be in the Hyperbola, and its ordinate *HN* ($= RP = DE = CT$) be equal to the Curve *ATt*. From whence, and from *Prop.* 3 *Coroll.* 2. of Dr. *Gregory's Catenaria* (*Phil. Tranſ.* N. 231.) it appears that the Curve *ATt* is that call'd the *Catenaria* or *Funicularia*, viz. the Curve into whoſe Figure a *ſlack Cord or Chain* naturally diſpoſes its ſelf by the Gravity of its Particles.

" 11. Hence we have another Property of the *Catenaria*
" not hitherto taken notice of (that I know of,) *viz.* that
" ſuppoſing *AR* ($= a$, the conſtant Line in Dr. *Gregory*)
" equal to the *Radius* of the Nautical Projection, and
" *RN* the Secant of a given Latitude, then is *NT* the
" *Catenaria's* Ordinate at *N*, equal to *RM* the Meridio-
" nal Part anſwering to the Latitude whoſe Secant is
" *RN*.

12. That *TA* is the *Catenaria* is alſo demonſtrable from Dr. *Gregory's* firſt *Prop.* Let *Tu* be the the Fluxion of the Ordinate *NT*; and *tu* ($= Nn$) the Fluxion of the Axe *AN*. Then becauſe of like Triangles *TCM*, *Tut*, $CM : CT (= TA) :: Tu : ut$, that is, as *CM* a conſtant Line to *TA* the Curve :: ſo is the Fluxion of the

 Ordi-

Ordinate, to that of the Axe ($\dot{y} : \dot{x}$) according to *Prop.* 1. *Catenariæ.*

13. From the Premises the Construction and several Properties of the *Catenaria* are easily deducible; one or two of which I'll set down.

1. The Area *ATMR* is equal to *AOPR* a Rectangle contained by Radius *AR* and *RP* the Tangent answering to Secant *HP* = *TM*. For because of the like Triangles *CMm*, *CEe*; *CM* : *CE* :: *Mm* : *Ee*, that is, putting *r*, *s*, *t*, *m* for Radius, Secant, Tangent and Meridional part *RM*.) $r : s :: \dot{m} : \dot{t}$ whence $r\dot{t} = s\dot{m}$, and all the $r\dot{t}$ = all the $s\dot{m}$, that is *AOPR* = *ATMR*, which agrees with Dr. *Gregory*'s *Cor.* 5. of *Prop.* 7.

14. Supposing the former Construction, let be added the Line *RH*, including the *Hyperbolic Sector ARH*. I say the same Sector is equal to half the Rectangle *ARMQ* contained by Radius *AR* and the Meridional Part *RM*, ($= \frac{1}{2} rm$), For the Sector *ARH* = Triangle *RNH* wanting the Semisegment *ANH*. The Fluxion of the Triangle *RNH* is $\frac{s\dot{t} + t\dot{s}}{2}$. The Fluxion of *ANH* is $t\dot{s}$. So the Fluxion of the Sector *ARH* is $\frac{s\dot{t} + t\dot{s}}{2} - t\dot{s} = \frac{s\dot{t} - t\dot{s}}{2}$. 'Tis found before (*Sect.* 13.) that $r : s \left(s : \frac{ss}{r}\right) :: \dot{m} : \dot{t}$; whence $s\dot{t} = \frac{ss}{r}\dot{m}$. And because of the like Triangles *CDE*, *Efe*, *CD* : *DE* :: *Ef* : *fe*. But $Ef = Mm = \dot{m}$, because both *Ef* and *Mm* are to *Md* in the same Reason, *viz.* as *s* to *r*; therefore $r : t \left(t : \frac{tt}{r}\right) :: \dot{m} : \dot{s}$: whence $t\dot{s} = \frac{tt}{r}\dot{m}$, and $\frac{s\dot{t} - t\dot{s}}{2} =$

2 ss

$\frac{ss - tt}{2r}\dot{m} = \frac{rr}{2r}\dot{m} = \frac{1}{2}r\dot{m}$, = the Fluxion of the Hyperbolic Sector *A R H*, whose flowing Quantity is therefore equal to $\frac{1}{2}rm = \frac{1}{2}$ *A R M Q. Q. E. D.*

15. This shews another Property of the *Catenaria*, viz. that it squares the Hyperbola; for *R M* is equal to *N T* the Ordinate of the *Catenaria*.

16. In *Fig.* 4. Let *A R* be Radius, *A C B* the Equitangential Curve; *M R N* its Asymptote, in which let *M*, *N*, be any two Points equally distant from *R*. Upon *M* draw *M L* parallel to *A R* and equal to the *Difference* of the Secant and Tangent of that Latitude whose Meridional Part is *R M* (by § 3, 4.) Upon *N* draw *N O* parallel to *A R*, and equal to the *Summ* of the foresaid Secant and Tangent. Do thus from as many Points in the Asymptote as is convenient, and a Curve drawn equably through the Points *L* - - - *A* - - - *O*, &c. will be a *Logarithmic Curve*, whose *Subtangent* (being constant) is equal to Radius *A R*.

17. Let *n o* be an Ordinate infinitely near and parallal to *N O*. *O p* = *N n* the Fluxion of the Asymptote; *O T* the Tangent, and *T N* the Subtangent to the Logarith. Curve in *O*. Then $op : pO :: ON : NT$. But $ON = s + t$; therefore $op = \dot{s} + \dot{t}$. $pO = \dot{m}$ (the Fluxion of the Meridian or Asymptote.) So the Analogy is $\dot{s} + \dot{t} : \dot{m} :: s + t : NT$. By *Sect.* 13, 14, $\dot{s} : \dot{m} :: t : r$. also, $\dot{t} : \dot{m} :: s : r$. and thence $\dot{s} + \dot{t} : \dot{m} :: t + s : r$. wherefore is *N T* (the Subtangent to *L A O*) equal to Radius *A R*, a constant Line, and consequently the Curve *L A O* is the Logarithmic Curve, and its Subtangent known.

18. The same Demonstration serves for *L M*, (any Ordinate on the other Side of *A R*) only changing the Sine + into —; and then it agrees with Mr. *James Gregory*'s *Prop.* 3. *pag.* 1?. of his *Exercitations*, viz. *That*

the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their respective Tangents, Radius being Unity. So here *R M* is the Logarithm of *M L*, the Difference of the Secant and Tangent of the Latitude whose Meridional part is *R M.*

19. Supposing the precedent Construction, if through any point *C* of the Curve *ACB* be drawn a right Line *G C W* parallel to *M R*, terminated with the Logarithmic Curve in *W* and the Radius *A R* in *G*: I say that the same right Line *WG* is equal to the intercepted part of the Curve Line *A C*.

20. Let *w g* be a Line infinitely near and parallel to *W G*, and terminated by the same Lines; and *C S*, *W σ*, perpendicular to the Meridian; *C S* intersecting *w g* in *z*, and *W σ* in *y*. Let *C M* be a Tangent to *A C* in *C*; *W τ* a Tangent to *A W* in *W*; so is *C M* = *σ τ*. Because of like Triangles *C z c*, *C S M*; and *W y w*, *W σ τ*; *C S* : *C M* :: *C z* : *C c*: also *W σ* : *σ τ* :: *W y* : *y w*. But *W σ* = *C S*; *σ τ* = *C M*; *C z* = *W y*; therefore is *y w* the Fluxion of *G W*, equal to *C c* the Fluxion of the Curve *A C*: Consequently *G W* = *A C*. *q. e. d.*

It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents standing perpendicular on their Radius. In *Fig.* 3. let *A γ Γ* be a Curve whose Ordinates as *g γ*, *G Γ*, are equal to the Tangents of their respective intercept Arcs *A k*, *A κ*. Let *Γ G* be produced to touch the Curve *A C* in *C*: then is the Area *A Γ G* equal to the Rectangle contained by Radius *A R* and *G C* the produced part of the Ordinate; or *A Γ G* = *A R* × *G C*. The Demonstration of which, and of the following *Section*, I for Brevity omit.

22. If we suppose the Figure *A C B* &c. *K R* (*Fig.* 3.) infinitely continued, to be turned about its Asymptote *R K* as an Axe, the Solid so generated will be equal to

a

rectangled Cone whose Altitude is equal to *A R*. And its Curve Surface will be equal to half the Surface of a Globe whose Radius is *A R*. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius *A R*.

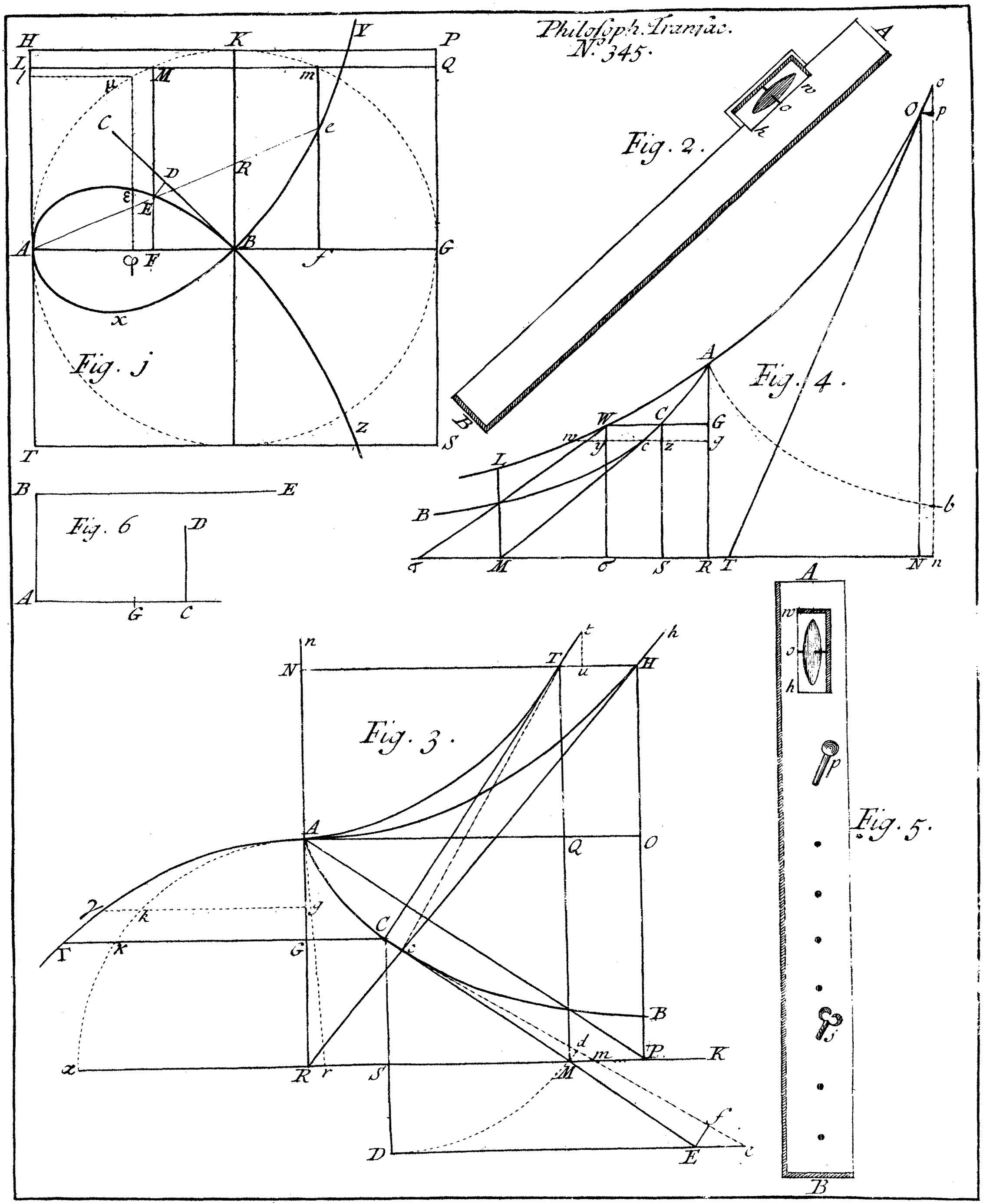
The Description of the Rular and Wheel, *Fig.* 2. is sufficient for the Demonstration of the Properties of the Curve: but in order to an actual Construction for Use, I have added *Fig.* 5. where *A B* is a brass Rular; *w h* the little Wheel, which must be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. *s* represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fixt Rular. *p* is a Stud for convenient holding the Rular in its Motion.

Note, *Most of these Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among those of the* Royal Academy *of* Sciences *for the Year* 1712. *but not publish'd till* 1715: *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their Journal.*

VI. *An Account of a Book entituled* Methodus Incrementorum, *Auctore* Brook Taylor, LL.D. & R. S. Secr. *By the Author.*

WHEN I apply'd my self to consider throughly the Nature of the Method of Fluxions, which has justly been the Occasion of so much Glory to its great Inventor Sir *Isaac Newton* our most worthy President, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatise. For it being the Foundation of the Method of Fluxions that the Flux-

ions


Philosoph. Transac.
N. 345.
Fig. j
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6